

A Fuzzy Set $A \subseteq \mathbb{R}^n$ (A_α α -cutset)
is convex if and only if

$$\mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$$

for all $x_1, x_2 \in \mathbb{R}^n$

الخط

Convex \iff الشرط يتحقق

\Rightarrow let A_α is Convex for all

$\Rightarrow \lambda x_1 + (1-\lambda)x_2 \in A_\alpha \quad 0 \leq \lambda \leq 1$

$\Rightarrow \mu_A(x_1) \geq \alpha ; \mu_A(x_2) \geq \alpha$

$$\therefore \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \alpha$$

$\Rightarrow \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \alpha = \min(\mu_A(x_1), \mu_A(x_2))$

if $\mu_A(x_1) \leq \mu_A(x_2)$

الاتجاه العكسي

let $\mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$

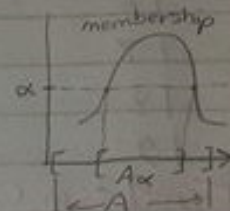
$$A_\alpha \neq \emptyset$$

If $\alpha = \mu_A(x_1) \leq \mu_A(x_2)$

$\Rightarrow \mu_A[\lambda x_1 + (1-\lambda)x_2] \geq \min[\alpha, \mu_A(x_2)]$

$\Rightarrow \lambda x_1 + (1-\lambda)x_2 \in A_\alpha$

$\therefore A_\alpha$ is convex



يصل عدد العناصر
متناهي $\leftarrow 1(\lambda) + 2(1-\lambda)$

$$0 \leq \lambda \leq 1$$

Magnitude of Fuzzy Set:

1] Scalar Cardinality:

يمثل هذا المقياس مجموع درجات الانتماء لجميع عناصر الفترة.

$$|A| = \sum_{x \in A} \mu_A(x)$$

2] Relative Cardinality:

$$||A|| = \frac{|A|}{|X|} = \frac{\sum_{x \in A} \mu_A(x)}{\text{no. of element of } X}$$

هذا المقياس يمثل مقدار التأكد من مصداقية ال data بالنسبة للصفة التي تجعل الفئة فئة فارغة

Ex: Consider the Fuzzy Set: Short, middle, tall

Cm	Short	Middle	tall
14	1	0	0
15	1	0	0
16	0.9	0.1	0
17	0.7	0.1	0
18	0.3	0.8	0.3
19	0	0	1

- Find:
- ① Compare the support of each set.
 - ② Compare the α -cut of each set at $\alpha=0.5$.
 - ③ $|short|$ and $||short||$

① $Supp(A) = \{x : \mu(x) > 0\}$
 $Supp(short) = \{14, 15, 16, 17, 18\}$
 $Supp(Middle) = \{16, 17, 18\}$
 $Supp(tall) = \{18, 19\}$

② $A_\alpha = \{x : \mu(x) > \alpha\}$
 $(Short)_{0.5} = \{14, 15, 16, 17\}$
 $(Middle)_{0.5} = \{17, 18\}$
 $(tall)_{0.5} = \{19\}$

③ $|short| = 1 + 1 + 0.9 + 0.7 + 0.3 = 3.9$
 $||short|| = \frac{3.9}{6}$

Operation on Fuzzy Set: العمليات على المجموعات الضبابية

① Complement:

$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \quad ; x \in X$

② Union:

$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$

③ Intersection:

$$\mu_{AB}(x) = \min[\mu_A(x), \mu_B(x)]$$

Ex: let $\tilde{A} = \frac{0.3}{1} + \frac{0}{2} + \frac{0.4}{3} + \frac{0.8}{4} + \frac{1}{5}$,

$\tilde{B} = \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.1}{3} + \frac{0.2}{4} + \frac{0.4}{5}$ Find:

① \tilde{A}

② $\tilde{A} \cup \tilde{B}$

③ $\tilde{A} \cap \tilde{B}$

④ $\tilde{A} - \tilde{B}$

⑤ $\tilde{A} \Delta \tilde{B}$

Sol ↓

① $\tilde{A} = \frac{0.7}{1} + \frac{1}{2} + \frac{0.6}{3} + \frac{0.2}{4} + \frac{0}{5}$

② $\tilde{A} \cup \tilde{B} = \frac{0.3}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0.8}{4} + \frac{0.1}{5}$

③ $\tilde{A} \cap \tilde{B} = \frac{0.2}{1} + \frac{0}{2} + \frac{0.1}{3} + \frac{0.2}{4} + \frac{0.4}{5}$

④ $\tilde{A} - \tilde{B} = \tilde{A} \cap \tilde{B}$

$\tilde{A} = \frac{0.3}{1} + \frac{0}{2} + \frac{0.4}{3} + \frac{0.8}{4} + \frac{1}{5}$

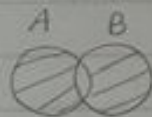
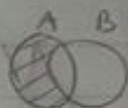
$\tilde{B} = \frac{0.8}{1} + \frac{0.7}{2} + \frac{0.9}{3} + \frac{0.8}{4} + \frac{0.6}{5}$

$\tilde{A} - \tilde{B} = \frac{0.3}{1} + \frac{0}{2} + \frac{0.4}{3} + \frac{0.8}{4} + \frac{0.4}{5}$

⑤ $\tilde{A} \Delta \tilde{B} = (\tilde{A} \cap \tilde{B}) \cup (\tilde{B} \cap \tilde{A})$

↓

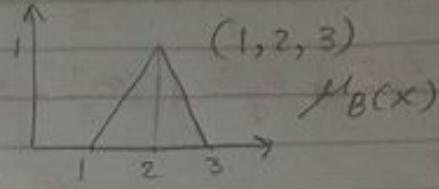
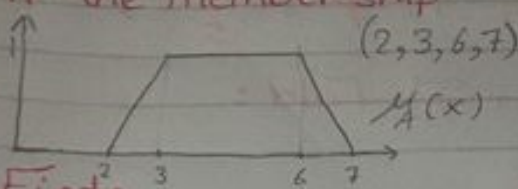
⋮



$A \Delta B = (A - B) \cup (B - A)$

في الحالة المنفصلة لا يوجد التكرارات المتعددة
discrete

Ex: Graphically represent the Fuzzy Set operation if the membership



Find:

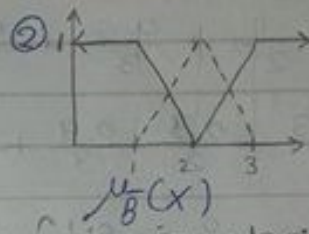
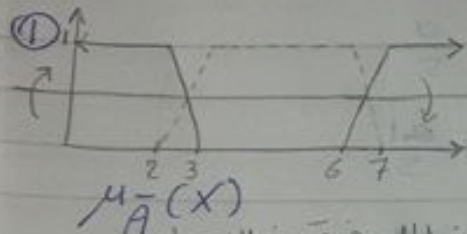
① $\mu_{\bar{A}}(x)$

② $\mu_{\bar{B}}(x)$

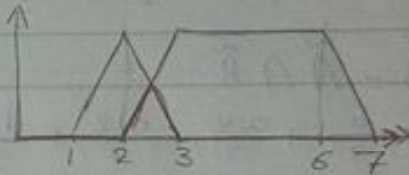
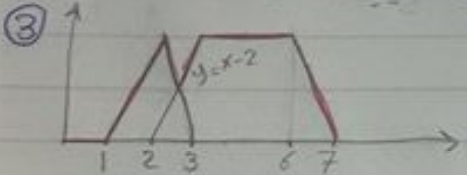
③ $\mu_{A \cup B}(x)$

④ $\mu_{A \cap B}(x)$

So ↓



③ نرسم الرسمين على بعض وننسى على الفرق الى فوق في الاتحاد



④ في التقاطع نرسم الرسمين على بعض وننسى على الفرق الى تحت